Project title: Quantum machine learning with Boson sampling

Supervisors: Stefan Scheel (German PI), Khabat Heshami (Canadian PI)

Current state of the art

Quantum physics, computing and machine learning are intrinsically linked together. Recent successes in experimental realizations of quantum computing devices have introduced the era of noisy intermediate-scale quantum computers (NISQ) that are not yet fully error-correctable, and only operate with few qubits. An alternative paradigm to quantum computing that are shown to be hard on classical computers are certain sampling algorithms, particularly those related to bosonic distributions [1]. The complexity results from the fact that computing the permanent, i.e. matrix elements in symmetrically ordered tensor product spaces, is computationally hard [2]. On the other hand, the permanent naturally occurs in linear optical networks [3, 4] (Fig. 1) and thus provides a connection to quantum physics.

Similar developments have taken place in machine learning, where kernel methods have been developed [5] that make use of the exponentially growing size of the Hilbert space as a function of the number of particles/modes, which can act as a feature space in which scalar products can be easily computed or measured. Recently, however, it has been found that if one increases the size of the system and the number of features, the variance in the fidelity kernel will decrease exponentially; reducing one's ability to effectively classify the input [6]. This exponential concentration means that an exponentially large number of measurements would have to be used for two states to be distinguished. However, they also argue that there would be a path for utilizing quantum effects more effectively to avoid this exponential concentration.

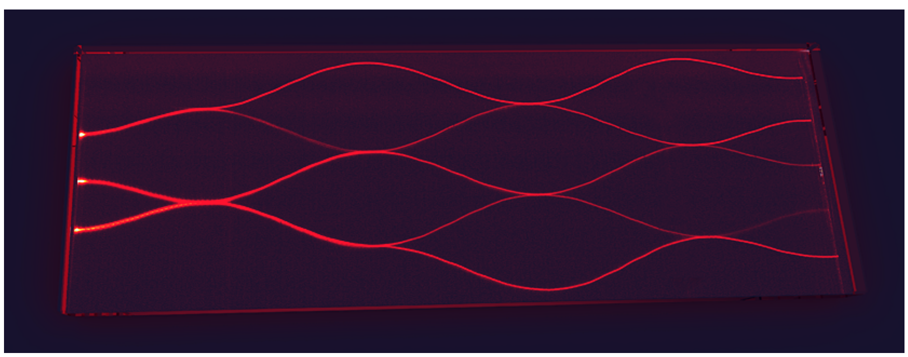

Figure 1: Linear optical network for use in boson sampling. Taken from Ref. [4].

Research goals and working program

The aim of this project is to study a variant of boson sampling using coherent states to use in kernel method machine learning. Despite the lack of a computational advantage compared to boson sampling, the permanent (and linear combinations thereof) still represent a mapping from a physical parameter space to a high-dimensional feature space, where the permanent replaces the scalar products in conventional kernel method machine learning. The aim is first to develop a method to use coherent-state sampling and coincidence measurements to gain access to (functions of) the permanent of a linear optical network. Next, we will be looking for ways how to implement machine learning protocols using these linear optical networks. We will be focusing on tunable networks that are increasingly becoming available, and that make use of thermal phase shifters inside Mach-Zehnder interferometers. An important aspect here will be to be able to show whether coherent-state sampling can compete will single-photon boson sampling. This requires to be able to prove detection sensitivities using permanent functions for which only few analytical properties are known [7]. One possibility is to devise a network that implements a particular unitary transformation, followed by another network that undoes most of that transformation (Fig. 2).

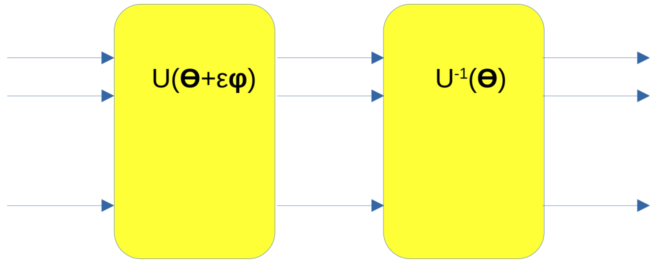

Figure 2: Sequence of linear optical networks that almost undo one another.

In this setting, we will follow up with the question of whether the effect of exponential concentration can be observed here as well, and on which physical parameters its appearance depends. This will guide us to choose alternative network designs that minimize the effect of exponential concentration.

The project supervisors, Prof. Stefan Scheel in Rostock and Prof. Khabat Heshami, have a long-standing expertise in theoretical quantum optics and quantum information. While the project is anchored in Rostock, the successful applicant is expected to spend several months (typically between 4-6 months) at the University of Ottawa.

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[4] M. Tillmann, B. Dakic, R. Heilmann, S. Nolte, A. Szameit, and P. Walther, *Experimental boson sampling*, Nature Photon. **7**, 540–544 (2013).

[5] M. Schuld and F. Petruccione, *Machine learning with quantum computers* (Springer, Cham, 2021).

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[7] H. Minc, *Permanents* (Addison-Wesley, Reading, 1978).